Energy Usage and Tire Usage in Electric Vehicles with In-Wheel Motors

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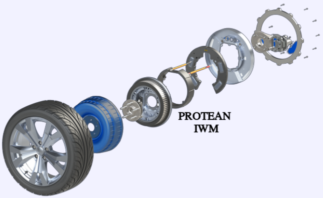
Outline

In-Wheel Motors (IWMs) and Battery Electric Vehicles (BEVs)

In-Wheel Motor (Motor seats insides the hub of a wheel)



- Mostly electronically commutated motors so-called **BLDC motors**, or **PMSMs**.
- Manufacturer: NTN, PROTEAN, ELAPHE, **AVID, MAGNAX**
- Axial flux IWMs: → Significantly higher torque and power densities

BEVs with single motor or one or multiple IWMs

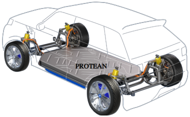
Bikes | **Microcars** | **Mid-size cars** | **Full-size cars** | **Buses**

Single motor drive (SMD)

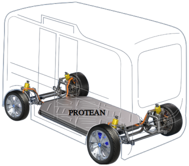
1 & 2 IWMs

2 & 4 IWMs [2WID & 4WID]

IWMs: Advantages and Challenges



- ☐ **Free up space inside EVs**
- ☐ **Better battery packaging**

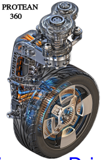


Challenges

- ☐ **High unprung mass**
- ☐ **Software complexities**
- ☐ **Prone to damage**
- ☐ **Expensive**

No transmission

Corner modules



[Steering + Driving + Braking + Suspension]

Overactuated EVs

- ☐ **Ease in manufacturing.**
- ☐ **Simplify assembly, repair, and maintenance processes**

Aim of the Thesis

Aim - 1

Battery Electric Vehicles (**BEVs**) →

- Mostly belong to **FIVE** types
- Lie in a **Wide Power** interval
- Use **Single, or One or Multiple IWMs**

■ Need to investigate these **BEVs** from Energy Usage Perspective.

Aim - 2

In-Wheel Motors (**IWMs**) → Allows make **overactuated EVs** having a potential to achieve **Optimal Tire Usage (OTU)**.

■ Need to investigate **overactuated EVs** from Tire Usage Perspective.

Outline

Motivation and Problem Formulation

Operational cost [Fuel Economy]   Environmental impact [Emission]

Questions:

1. How energy efficient are these BEVs?

2. How energy efficient are they in comparison to ICE vehicles?

Energy Efficiency

Battery Electric Vehicles (BEVs)

SMD BEVs

IWM BEVs

1IWM BEVs (Bikes) | 2IWM BEVs | 4IWM BEVs

Wide power interval

Bikes	Microcars	Mid-size cars	Full-size cars	Buses
[0.25 – 8]kW	[8 – 16]kW	[16 – 60]kW	[60 – 160]kW	[80 – 360]kW

Why and How?

Environmental issues → BEVs

Government policies → BEVs

Emerging technologies (Motor, Battery, and so on) → BEVs

BEVs

↓

Energy Efficiency {**Well-to-Wheels (WTW)**}

- **Well-to-Tank (WTT)** {To compare all vehicles}
- **Tank-to-Wheels (TTW)** {To compare among BEVs}

1) OPTIRESOURCE Software → Energy Efficiency

- Emphasizes WTW analysis
- Emission + fuel economy
- 1000+ energy pathways
- 12 generic vehicles (1 BEV)
- Uses experimental data
- Does not show model
- **Not applicable for all BEVs (TTW)**
- **Does not reveal η_{TTW}**

2) Thesis analysis (10 other works) → Energy Efficiency

- Emphasizes TTW analysis
- No emission, η_{TTW} (%)
- Natural gas pathways
- 15 BEVs
- Uses data from literature
- Shows model (stages)
  - **Applicable for all BEVs**
  - **Does not reveal fuel economy**

Literature Review

The existing works → **Energy efficieny of BEVs** → **Involves energy consuming stages.** **[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]**

- → A **generic**, or **Nissan Leaf, Peugeot Ion, Tesla Roadster**
- → **Mid-size or full-size** cars
- → **Non-IWM BEVs**, except one work.
- → Variety in **number of stages.**
- → Variety in operating conditions, for example, **drive cycles**.

Unanswered questions?

- → Q1: What is TTW energy efficiency (η_{TTW}) of IWM BEVs of different values of power?
- → Q2: $\eta_{\mathrm{TTW}}^{\mathrm{IWM}}$(BEVs) > or < $\eta_{\mathrm{TTW}}^{\mathrm{SMD}}$ (BEVs) at various values of power?
- → Q3: Trends of $\eta_{\mathrm{TTW}}^{\mathrm{IWM}}$(BEVs) and $\eta_{\mathrm{TTW}}^{\mathrm{SMD}}$ (BEVs) depending on vehicle power?
- → Q4: $\eta_{\mathrm{TTW}}^{\mathrm{4WID}} : \eta_{\mathrm{TTW}}^{\mathrm{2WID}} : \eta_{\mathrm{TTW}}^{\mathrm{SMD}}$ at each values of power and by how much?

The need:

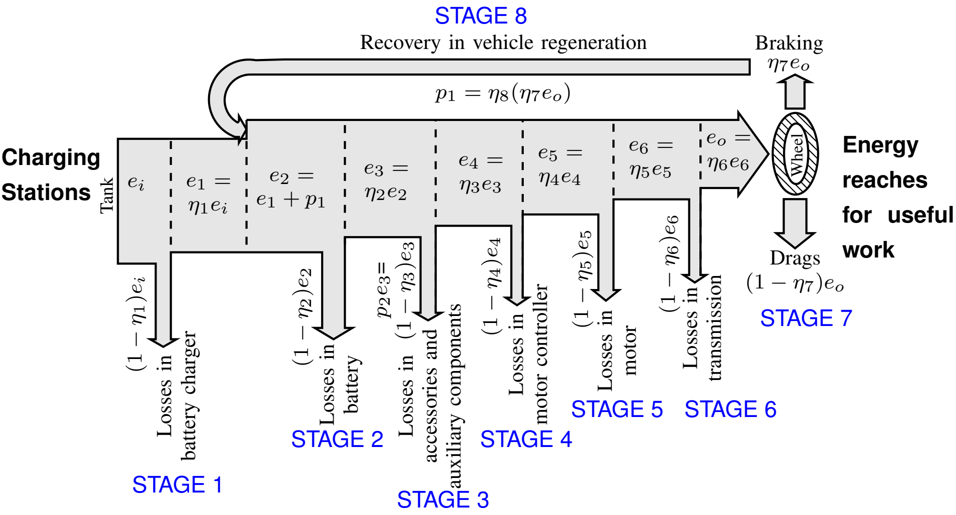
1. A methodology common to all BEVs

Answer:

1. A graph of η_{TTW} of BEVs vs. their powers.

Methodology

■ Generalized Energy Flow Diagram



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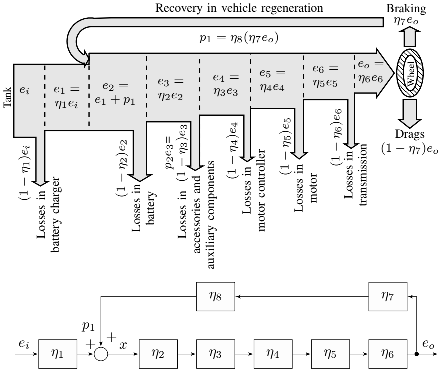
■ Generalized Energy Flow Diagram



For k-th iterations on drive cycle:

$e_o(k \rightarrow \infty) = \frac{\eta_1 \cdot \eta_f}{1 - \eta_{fb} \cdot \eta_f} \cdot e_i;$

Where, $\eta_f = \eta_2 \eta_3 \eta_4 \eta_5 \eta_6$,

$\eta_{fb} = \eta_7 \eta_8$, $0 < \eta_{fb}, \eta_f < 1$,

and e_i=100 for TTW in %.

■ Facts and Assumption

F1: **Lithium-based battery BEVs** constitute an overwhelming majority of BEVs.

F2: IWM and SMD BEVs widely use **permanent magnet motors**, such as PMSMs and EC motors.

F3: SMD BEVs use **single-stage-gear mechanical transmission**.

A1: BEVs use **EPA (FTP) urban drive cycle**.

FTP: Federal Test Procedure

Calculation of TTW Energy Efficiency of BEVs

Equation for calculating η_{TTW}

$$\eta_{\mathrm{TTW}} = \frac{\eta_1 \cdot \eta_f}{1-\eta_{fb}\cdot\eta_f} \cdot e_i;\ \text{when } e_i = 100 \text{ units}$$

Table to find η_{TTW} of 15 kinds of BEVs: SMD, 2IWM, 1IWM, 2WID, 4WID BEVs

Vehicle type	BEVs	Power range [kW]	η_1 (in %)	η_2 (in %) F1	η_3=$(1-p_2)$ (in %), A1	η_4 (in %)	η_5 (in %), F2	η_6 (in %), F3	η_7 (in %), A1	η_8 (in %), A1	e_o (η_{TTW})
Bikes	2IWM	2×(0.25 – 4)	88.0	87.0	95	95.8 – 97.1	81.2 – 90.7	100	**32**	**35**	61.0 – 71.5
	1IWM	0.25 – 8	88.0	87.0	95	95.8 – 97.4	81.2 – 92.5	100	**28**	**23**	59.0 – 67.2
	SMD	0.25 – 8	88.0	87.0	95	95.8 – 97.4	81.2 – 92.5	98	**28**	**22**	57.7 – 67.2
Microcars	4WID	4×(2 – 4)	91.0	87.0	74	96.8 – 97.1	88.7 – 90.7	100	**30**	**45**	54.4 – 55.9
	2WID	2×(4 – 8)	91.0	87.0	74	97.1 – 97.4	90.7 – 92.5	100	**29**	**29**	54.4 – 55.5
	SMD	8 – 16	91.0	87.0	74	97.4 – 97.7	92.5 – 94.0	95	**29**	**28**	52.5 – 53.6
Mid-size cars	4WID	4×(4 – 15)	92.0	87.0	72	97.1 – 97.7	90.7 – 93.9	100	**36**	**55**	57.0 – 59.7
	2WID	2×(8 – 30)	92.0	87.0	72	97.4 – 97.9	92.5 – 95.0	100	**35**	**36**	55.9 – 57.9
	SMD	16 – 60	92.0	87.0	72	97.7 – 98.1	94.0 – 95.8	95	**36**	**34**	53.9 – 55.2
Full-size cars	4WID	4×(15 – 40)	93.0	87.0	72	97.7 – 98.0	93.9 – 95.4	100	**41**	**65**	63.1 – 64.6
	2WID	2×(30 – 80)	93.0	87.0	72	97.9 – 98.2	95.0 – 96.0	100	**40**	**42**	60.1 – 61.0
	SMD	60 – 160	93.0	87.0	72	98.1 – 98.2	95.8 – 96.2	95	**41**	**40**	57.2– 57.5
Electric buses	4WID	4×(20 – 90)	93.0	87.0	71	97.8 – 98.2	94.4 – 96.0	100	**51**	**75**	68.0 – 69.8
	2WID	2×(40 – 180)	93.0	87.0	71	98.0 – 98.2	95.4 – 96.2	100	**50**	**50**	62.6 – 63.4
	SMD	80 – 360	93.0	87.0	71	98.2 – 98.3	96.0 – 96.2	95	**51**	**47**	59.3 – 59.5

BEVs and their variants | Motor Powers | 8 STAGES | η_{TTW}

STAGE 1 & 2

STAGE 1: Battery Charger (η_1)

Vehicle type	Battery energy capacity (in kWh)	Recommended charging times (in hours)	Recommended charger powers (in kW)	Average charger efficiency, η_1 (in %)
Bikes	1 – 4	1 – 8	0.5 – 1.2, 1.4, 1.8, 2.3	88.0
Microcars	4 – 14	2 – 8	1.8, 2.3, 3.3, 3.7	91.0
Mid-size cars	16 – 30	2 – 10	3.3, 3.7, 7.4, 7.7, 11	92.0
Full-size cars	30 – 100	3 – 12	7.4, 7.7, 11.0, 17.3, 22.0	93.0
Electric buses	53 – 550	1 – 12	11.0, 22.0, 50.0	93.0,

■ Considerations:

(1) Selected suitable battery capacities

(2) Selected suitable charging times

(3) Selected recommended chargers

■ Trend: η_1 increases with charger power, and saturates at high powers.

STAGE 2: Battery (η_2)

□ **Lithium-based battery (LFP, Lithium-ion, Lithium polymer)**

- Most promising and dominant EV battery technology
- High energy and power density
- Facilitating fast charging
- The safest and non-toxicity (LFP)

□ Average efficiency of lithium-based cells=90%

□ **Battery management system** ≈ 3 % from the battery pack

- Cell balancing (1 – 2 %)
- Monitoring, protection & cooling(1 – 2 %)

□ Thus average efficiency of battery ≈ 87 %

■ Trend: η_2 is roughly the same for all BEVs.

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STAGES 3, 4, 5 & 6

STAGE 3: Losses in Accessories and Auxiliary Electrical Components (p_2)

↳ **The loss is a fraction of battery energy.**

□ Depends on drive-cycle

□ Data available for **Nissan Leaf** ≈ 28%

□ This work assumes (by providing proper justification) that

–Bikes ≈ 5%

–Microcars ≈ 26%

–Mid-size cars ≈ 28%

–Buses ≈ 29%

■ Trend: $\eta_3 = 1 - p_2$ decreases with BEV power in the case of four-wheel BEVs.

STAGE 6: Mechanical Transmission

□ IWM BEVs: No mechanical transmssion

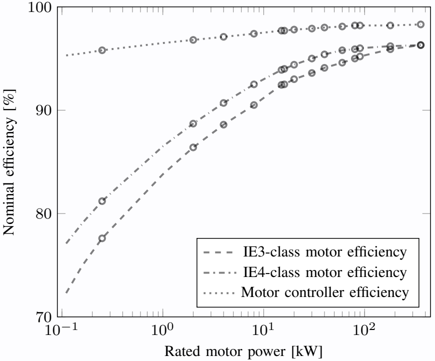
□ SMD bikes → Mid-drive bikes → Chain or belt drive → about 98% efficient.

□ SMD cars and buses → Single-stage gear transmission → about 95% efficient.

■ Trend: η_6 is roughly constant.

STAGE 4 & 5: Motor Controller (η_4) and Motor (η_5)

■ Efficiency of permanent magnet (PM) motors are close IE-4 class of motors



□ **Efficiency of 4IWMs < Efficiency of a single motor of equivalent power**

■ Trend:

η_4 and η_5 increase with BEV power.

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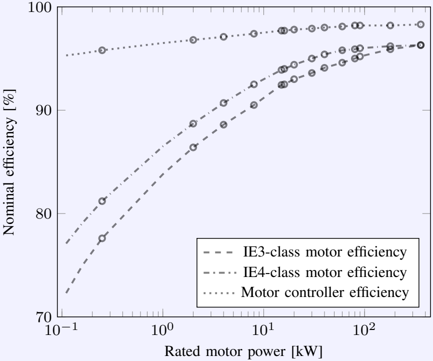

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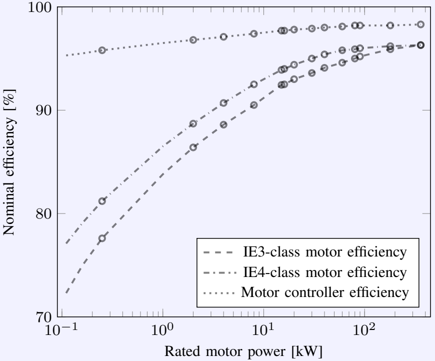

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■ Trend:

η_4 and η_5 increase with BEV power.

STAGES 7 & 8

STAGE 7: Braking (η_7)

↳ **Fraction of TTW energy that goes into braking**

- Drive-cycle dependent
- TTW energy → Drags(aero+rolling resistance) / → **Braking (Recoverable)**

Thanks to Mr. Vasu Jain

- $\eta_7 = \frac{e_{br}}{e_o} = \frac{\gamma\cdot(M_e/M)-[\alpha'\cdot r_o+\beta'\cdot(C_D\cdot A/M)]}{\gamma\cdot(M_e/M)+[\alpha\cdot r_o+\beta\cdot(C_D\cdot A/M)]}$ [11]

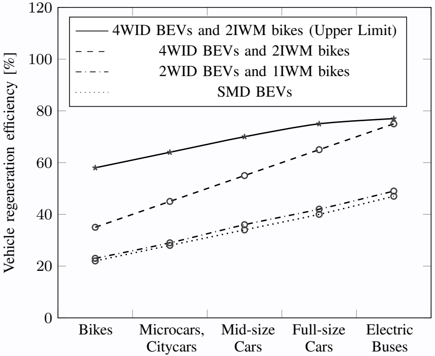
Vehicle type	BEVs	r_o	C_D	$A[\mathrm{m}^2]$	M[kg]	M_e/M	$\eta_7(\%) = \frac{e_{br}}{e_o}\cdot 100$
Microcars	4WID	0.013	0.35	1.8	630	1.04	30
	2WID	0.013	0.35	1.8	570	1.04	29
	SMD	0.013	0.35	1.8	600	1.04	29
Mid-size cars	4WID	0.013	0.30	2.0	1050	1.04	36
	2WID	0.013	0.30	2.0	950	1.04	35
	SMD	0.013	0.30	2.0	1000	1.04	36
Full-size cars	4WID	0.013	0.28	2.1	1680	1.04	41
	2WID	0.013	0.28	2.1	1520	1.04	40
	SMD	0.013	0.28	2.1	1600	1.04	41
Electric buses	4WID	0.008	0.65	7.0	12600	1.04	51
	2WID	0.008	0.65	7.0	11400	1.04	50
	SMD	0.008	0.65	7.0	12000	1.04	51

- Depends on **vehicle mass, effective mass, and drag and rolling resistance coefficients.**

■ Trend: η_7 increases with BEV power.

STAGE 8: Vehicle Regeneration (η_8)

■ Estimated from available data in the literature



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	2WID	0.013	0.30	2.0	950	1.04	35
	SMD	0.013	0.30	2.0	1000	1.04	36
Full-size cars	4WID	0.013	0.28	2.1	1680	1.04	41
	2WID	0.013	0.28	2.1	1520	1.04	40
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Electric buses	4WID	0.008	0.65	7.0	12600	1.04	51
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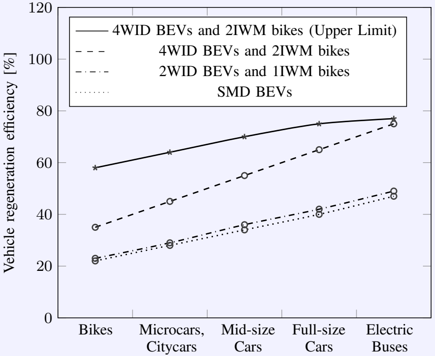
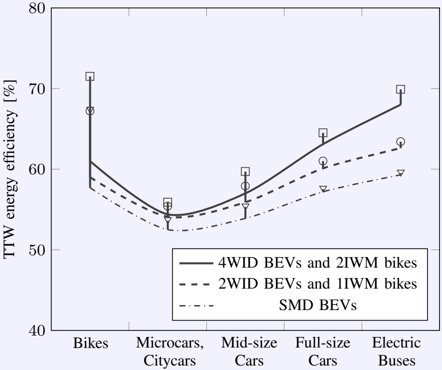


■ Trend: Vehicle regeneration efficiency increases with vehicle power.

Results and Conclusion

Graph of η_{TTW} versus BEV power



- ☐ IWM mid- and full-size cars: η_{TTW} = [56 – 65%]
- ☐ SMD mid- and full-size cars: η_{TTW} = [54 – 58%]
- ☐ $\eta_{\mathrm{TTW}}^{\mathrm{SMD}} < \eta_{\mathrm{TTW}}^{\mathrm{4WID}}$ by upto 18 %.

■ In the case of **four-wheeled BEVs:**
- ☐ η_{TTW} increases with BEV power.
- ☐ $\eta_{\mathrm{TTW}}^{\mathrm{4WID}} > \eta_{\mathrm{TTW}}^{\mathrm{2WID}} > \eta_{\mathrm{TTW}}^{\mathrm{SMD}}$

Trends in Stages (η_1, ..., η_8)

■ In the case of **four-wheeled BEVs:**
- ☐ Five stages (η_1, η_4, η_5, η_7, and η_8): Increase with BEV power.
- ☐ Two stages (η_2 and η_6): Constant with BEV powers.
- ☐ One stage (η_3): Decreases (insignificantly) with BEV powers.

■ The observed qualitative trends are:
- ☐ η_{TTW} increases with BEV power.
- ☐ $\eta_{\mathrm{TTW}}^{\mathrm{4WID}} > \eta_{\mathrm{TTW}}^{\mathrm{2WID}} > \eta_{\mathrm{TTW}}^{\mathrm{SMD}}$ in higher power range.

Correctness of Results

■ IWM mid-size or full-size cars:

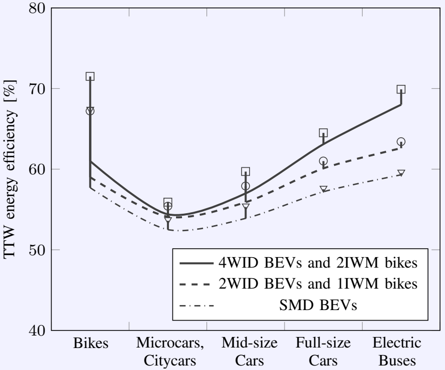
η_{TTW}=[64 – 68%] (One publication)

■ SMD mid-size and full-size cars:

η_{TTW}=[51 – 71%] (Six publications)

Results and Conclusion

Graph of η_{TTW} versus BEV power



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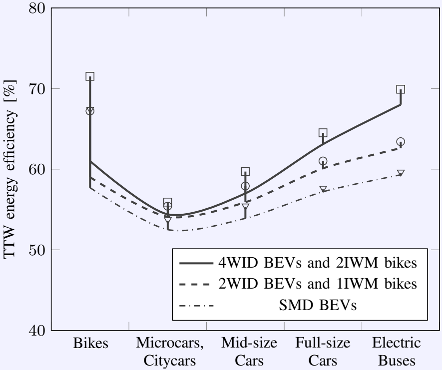
Correctness of Results

■ IWM mid-size or full-size cars:
η_{TTW}=[64 – 68%] (One publication)
■ SMD mid-size and full-size cars:
η_{TTW}=[51 – 71%] (Six publications)

Results and Conclusion

Graph of η_{TTW} versus BEV power



- ☐ IWM mid- and full-size cars: η_{TTW} = [56 – 65%]
- ☐ SMD mid- and full-size cars: η_{TTW} = [54 – 58%]
- ☐ $\eta_{TTW}^{SMD} < \eta_{TTW}^{4WID}$ by upto 18 %.

■ In the case of **four-wheeled BEVs:**
- ☐ η_{TTW} increases with BEV power.
- ☐ $\eta_{TTW}^{4WID} > \eta_{TTW}^{2WID} > \eta_{TTW}^{SMD}$

Trends in Stages (η_1, ..., η_8)

■ In the case of **four-wheeled BEVs:**

☐ Five stages (η_1, η_4, η_5, η_7, and η_8): Increase with BEV power.

☐ Two stages (η_2 and η_6): Constant with BEV powers.

☐ One stage (η_3): Decreases (insignificantly) with BEV powers.

■ The observed qualitative trends are:

☐ η_{TTW} increases with BEV power.
☐ $\eta_{TTW}^{4WID} > \eta_{TTW}^{2WID} > \eta_{TTW}^{SMD}$ in higher power range.

Correctness of Results

■ IWM mid-size or full-size cars:

η_{TTW}=[64 – 68%] (One publication)
■ SMD mid-size and full-size cars:

η_{TTW}=[51 – 71%] (Six publications)

Results and Conclusion

η_{TTW} of a BEV and Impact of Mass

■ **Consider a full-size SMD car with**

$M = 1400$ kg, $M_e/M = 1.05$, $A = 2.3\ \text{m}^2$, $r_o = 0.01$, $C_D = 0.30$. $\implies \eta_7 = 44.0\%$

For $\eta_1 = 93.0$, $\eta_2 = 92.0\%$, $\eta_3 = 70\%$, $\eta_4 = 98.0\%$, $\eta_5 = 96.0\%$, $\eta_6 = 97.0\%$, $\eta_8 = 45\%$.

$\implies \eta_{\text{TTW}} = 62.2\ \%$.

■ **Impact on η_{TTW} with mass change (ΔM):**

When $\Delta M = 10\% \implies \Delta\eta_{\text{TTW}} \approx 0.3\ \%$

When $\Delta M = 100\% \implies \Delta\eta_{\text{TTW}} \approx 1.0\ \%$

Issues in Including of Margin within η_{TTW}

■ **Data is sufficiently not available**

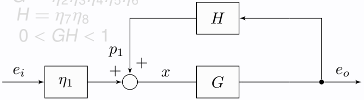
■ **Taking 2% margin within each stage** $\implies$ **7 – 10% margin in η_{TTW} of each BEV.**

□ Does not answer the questions raised.

□ Battery Charger (η_1), Battery (η_2), Auxiliary losses (η_3), and Transmission (η_6): Common (Worst/best-cases are applicable to all BEVs)

Recommendations for Improving η_{TTW}

$G = \eta_2\eta_3\eta_4\eta_5\eta_6$
$H = \eta_7\eta_8$
$0 < GH < 1$



#1. $\eta_{\text{TTW}} = \frac{e_o}{e_i} = \frac{\eta_1 G}{1-GH} = \eta_1 G(1 + GH + G^2H^2 + ...)$

□ The higher the H, G, or GH, the higher the η_{TTW}.

#2. Let $S_G^{\eta_{\text{TTW}}} = \frac{\partial\eta_{\text{TTW}}}{\eta_{\text{TTW}}} / \frac{\partial G}{G}$ and $S_H^{\eta_{\text{TTW}}} = \frac{\partial\eta_{\text{TTW}}}{\eta_{\text{TTW}}} / \frac{\partial H}{H}$

$$S_G^{\eta_{\text{TTW}}} > S_H^{\eta_{\text{TTW}}}$$

□ Higher ΔG is more helpful in improving η_{TTW}.

□ $\Delta H > \Delta G$ for ΔH to make greater change in η_{TTW}.

#3. $\Delta H > \Delta G$, how much greater?

$$\frac{\Delta H}{\Delta G} > \frac{1}{G^2} \quad \text{(a)}$$

■ For 2WID and 4WID BEVs of the same power class.

$$\Delta H(= H_{\text{4WID}} - H_{\text{2WID}}) \gg \Delta G(= G_{\text{4WID}} - G_{\text{2WID}} \approx 0)$$

$\eta_{\text{TTW}}^{\text{4WID}} > \eta_{\text{TTW}}^{\text{2WID}}$ [(a) is satisfied]

■ For SMD and 2WID BEVs of the same power class.

$$\Delta H(= H_{\text{2WID}} - H_{\text{SMD}}) > \Delta G(= G_{\text{2WID}} - G_{\text{SMD}})$$

$\eta_{\text{TTW}}^{\text{2WID}} > \eta_{\text{TTW}}^{\text{SMD}}$ [(a) is not satisfied, but $G_{\text{2WID}} \gg G_{\text{SMD}}$]

Results and Conclusion

η_{TTW} of a BEV and Impact of Mass

■ **Consider a full-size SMD car with**

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$\Longrightarrow$ 7 – 10% **margin in η_{TTW} of each BEV.**

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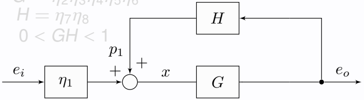

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$\eta_{\text{TTW}}^{\text{4WID}} > \eta_{\text{TTW}}^{\text{2WID}}$ [(a) is satisfied]

■ For SMD and 2WID BEVs of the same power class.

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$\eta_{\text{TTW}}^{\text{2WID}} > \eta_{\text{TTW}}^{\text{SMD}}$ [(a) is not satisfied, but $G_{\text{2WID}} \gg G_{\text{SMD}}$]

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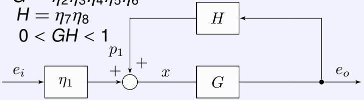

#1. $\eta_{\text{TTW}} = \frac{e_o}{e_i} = \frac{\eta_1 G}{1-GH} = \eta_1 G(1 + GH + G^2H^2 + ...)$

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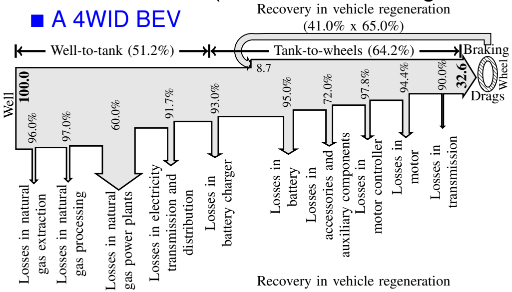
$\eta_{\text{TTW}}^{\text{2WID}} > \eta_{\text{TTW}}^{\text{SMD}}$ [(a) is not satisfied, but $G_{\text{2WID}} \gg G_{\text{SMD}}$]

Results and Conclusion

■ WTW efficiency of a 4WID BEV, an SMD BEV, and an ICE vehicle

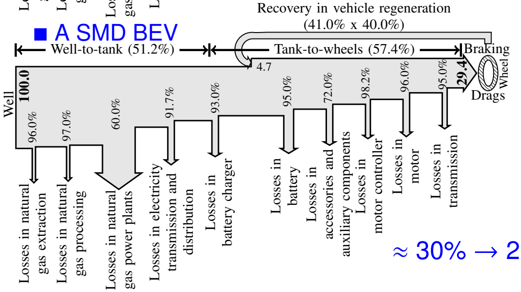
(Full-size cars using **Natural gas pathway.**)

■ A 4WID BEV



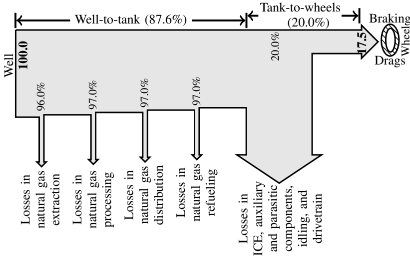
≈ 33% → 29% (In very cold condition)

■ A SMD BEV



≈ 30% → 26% (In very cold condition)

■ An Internal Combustion Engine Vehicle



≈ 18% → 24%

(In very cold condition)

Key Findings and Usefulnesses

Key findings:

- Identification of important and unanswered questions in the existing literature.
- A generalized EFD to calculate η_{TTW} of BEVs.
- A graph of η_{TTW} versus BEV power.
- An analysis for improving η_{TTW} of BEVs.
- Qualitative trends in η_{TTW} of BEVs.
- WTW efficiencies of 4WID, SMD and ICE full-size cars.
- The TTW analysis is a complementary to 'OPTIRESOURCE' (a software tool).

Usefullnesses:

- To determine η_{TTW} of any BEV.
- For users, like fuel economy η_{TTW} can also be performance indicators of BEVs.
- For manufacturers to improve η_{TTW} of their BEVs.
- For industries to invest in new BEV technologies.
- For manufacturers to find impact of variation in vehicle mass η_{TTW}.
- For industries, policy makers, governments, and analysts in evaluating environmental, social, and governance (ESG) performances of all kinds of EVs.

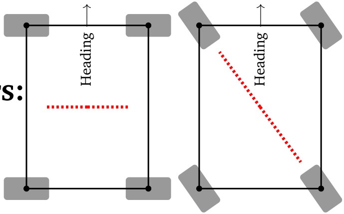
Outline

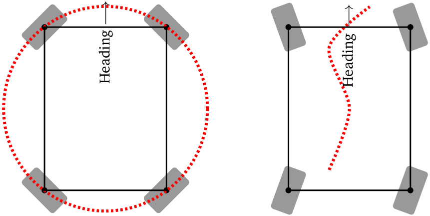
Introduction and Motivation

The thesis primarily considers
4WIS4WID EVs

Four steering motors | Four driving motors, mostly IWMs

Manoeuvers:





Source: 4WIS4WID EV by DLR (Germany).

- **Prominant Institutions:** MIT, Stanford, Otago Polytechnic, KTH, Endhovan University, DLR, University of Texas, IIT Kanpur.
- Have a potential to achieve optimal tire usage (OTU)

What is Optimal Tire Usage (OTU)?

■ Condition for OTU [13]:

$$\mathcal{K}_1 = \mathcal{K}_2 = \mathcal{K}_3 = \mathcal{K}_4 = \mathcal{K}_{\min}$$

□ Tire friction usage, $\mathcal{K}_i$ of i-th wheel

$$\mathcal{K}_i = \frac{\sqrt{F_{xti}^2 + F_{yti}^2}}{\mu_{\max} F_{zi}} = \frac{\sqrt{F_{xli}^2 + F_{yli}^2}}{\mu_{\max} F_{zi}}$$

□ Constraints:

$$m(\dot{v}_{xl} - \gamma v_{yl}) = F_{xl} = F_{x1} + F_{x2} + F_{x3} + F_{x4} \quad \text{(1a)}$$

$$m(\dot{v}_{yl} + \gamma v_{xl}) = F_{yl} = F_{y1} + F_{y2} + F_{y3} + F_{y4} \quad \text{(1b)}$$

$$I_z \dot{\gamma} = M_{zl} = l_d(-F_{x1} + F_{x2} - F_{x3} + F_{x4}) + l_f(F_{y1} + F_{y2}) - l_r(F_{y3} + F_{y4}) \quad \text{(1c)}$$

- No tire reaches saturation before others.
- Largest gap between operating and saturation points of tires.

■ Tire-grip margin, $\mathcal{G}_i$ of i-th wheel,

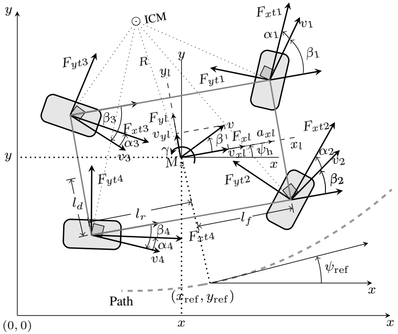
$$\mathcal{G}_i = 1 - \mathcal{K}_i$$

■ Vehicle stability margin, $\mathcal{S}$

$$\mathcal{S} = \min\{\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3, \mathcal{G}_4\}$$

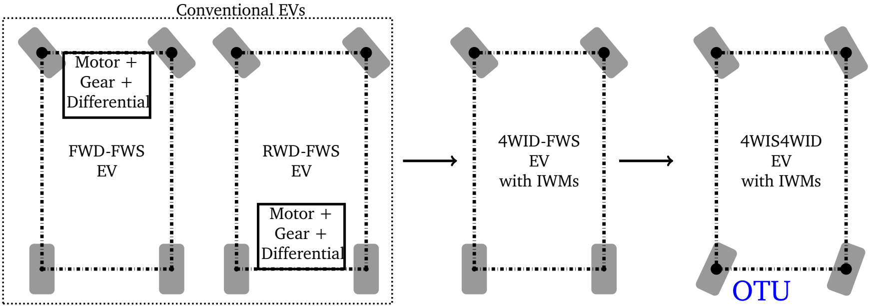
□ With OTU

$\mathcal{S} = 1 - \mathcal{K}_{\min}$ **(Superior $\mathcal{S}$)**



[13] Hyungchai Park and J. Christian Gerdes. "Optimal tire force allocation for trajectory tracking with an over-actuated vehicle." 2015 IEEE Intelligent Vehicles Symposium (IV). IEEE, 2015.

Motivation and Importance of OTU



Increase in TTW Energy Efficiency → upto 18%[14] → To some extent

Considering all energy consuming stages

Improvement

Due to reduced tire slip losses

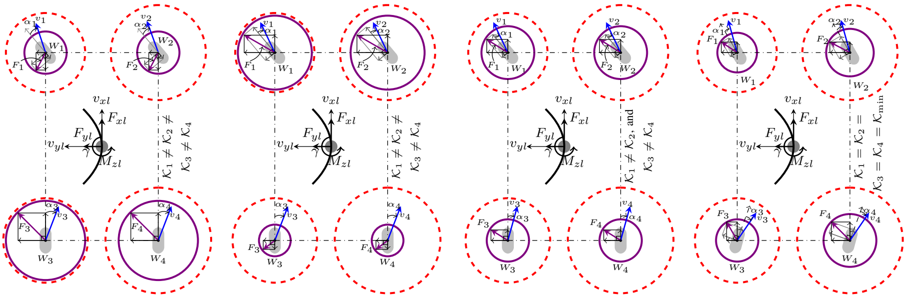
Increase in vehicle stability margin → Superior vehicle stability margin

Due to improved tire grip margin

- Acceleration and braking performance
- Controllability and stability
- Better cornering behaviour
- More responsive
- Improved safety critical situations

[14] Dileep Kumar, Vasu Jain, and Ramprasad Potluri, "Energy efficiency of battery electric vehicles with in-wheel motors," SAE International Journal of Sustainable Transportation, Energy, Environment, & Policy, vol. 4, no. 13-04-01-0002, 2022.

An Example to Compare Tire Usages in four EVs



(a) Front-wheel steering and rear-wheel drive EV

(b) Front-wheel steering and front-wheel drive EV

(c) Front-wheel steering and all-wheel drive EV

(d) Four-wheel steering and Four-wheel drive EV

Vehicle type	(a) Front-wheel steering and rear-wheel drive EV	(b) Front-wheel steering and front-wheel drive EV	(c) Front-wheel steering and all-wheel drive EV	(d) Four-wheel steering and Four-wheel drive EV
Tire friction usage	– W_3 is overutilized – W_2 is underutilized. – $K_3 > K_4 > K_1 > K_2$ – W_3 decides the vehicle stability margin $\mathcal{S}$.	– W_1 is overutilized – W_4 is underutilized. – $K_1 > K_2 > K_3 > K_4$ – W_1 decides the vehicle stability margin $\mathcal{S}$.	– W_1 is overutilized – W_4 is underutilized. – $K_1 > K_2$ and $K_3 > K_4$ – W_1 decides the vehicle stability margin $\mathcal{S}$.	– All wheels are equally and minimally utilized – $\mathcal{K}_1 = \mathcal{K}_2 = \mathcal{K}_3 = \mathcal{K}_4$ – All wheels decide the vehicle stability margin.
Stability Margin	Low		Moderately high	High

☐ A vehicle can be stable without OTU.

However, it is more prone to instability without OTU than with OTU.

Literature Review and Problem Identifications

About 15 works on PTC algorithms of 4WIS4WID EVs

Master control level [Control Allocation] ⟷ Full PTC algorithm ⟷ Slave control level [Motor and MCSs]

10 works aspire for OTU explicitly or implicitly

FCA Approach [Optimization]

- 2 work → Mimimization of $\mathcal{K}_i$ and constrained by $\mathcal{K}_1 = \mathcal{K}_2 = \mathcal{K}_3 = \mathcal{K}_4$ (1 work → Show Full PTC Alogrithms for 4WIS4WID EVs)
- 1 work → Indicate achievement of OTU [Illustrations] (1 work → Show Full PTC Alogrithms for 4WIS4WID EVs)
- 5 works → Indicate achievement of OTU approximately [Illustrations, or formulation of cost of function] (3 work → Show Full PTC Alogrithms for 4WIS4WID EVs)

KA approach [Kinematics]

- 1 work → Indicate achievement of OTU approximately and **implicitly** (1 work → Show Full PTC Alogrithms for 4WIS4WID EVs)
- 1 work → Report achievement of OTU approximately and **implicitly** (1 work → Show Full PTC Alogrithms for 4WIS4WID EVs)

Show Full PTC Alogrithms for 4WIS4WID EVs → 2 KA works, 3 FCA works → In-depth Block-Diagrammatic Examination

Why implicit?

1. To see whether these PTC alorithms have all required elements for OTU.

2. To suggest modifications, if needed.

Continued...

Part I: How to check OTU in the two KA works, and what is extent they can achieve OTU?

The 5 selected works

No optimization at master control level (cannot claim OTU)

KA approach

- ICM allocation → Potluri et al. (IITK) [15]
- Position allocation → Ploeg et al. (TNO Netherlands) [16]

Optimization at master control level (can claim OTU)

FCA approach

- Tire force allocation → Castro et al. (DLR) [18]
- Tire force allocation → Park et al. (Stanford university) [13]
- Wheel slip allocation → Leenen (Eindhoven University of Technology) [17]

Part II: Block diagrammatic examinations to identify modifications and suggest appropriate solutions

[15]: R. Potluri and A.K. Singh. Path-Tracking Control of an Autonomous 4WS4WD Electric Vehicle using its Natural Feedback Loops. IEEE Transactions on Control Systems Technology, 23(5):2053–2062, 2015.
[16]: Jeroen Ploeg, Hanno E Schouten, and Henk Nijmeijer. Position control of a wheeled mobile robot including tire behavior. IEEE Transactions on Intelligent Transportation Systems, 10(3):523–533, 2009.
[17]: Roe Leenen, J Ploeg, H Nijmeijer, L Moreau, and F Veldpaus. Motion control design for a 4ws and 4wd overactuated vehicle. Master's thesis, Department Mechanical Engineering Dynamics and Control Technology Group, Eindhoven University of Technology, Eindhoven, 2004.
[13]: Hyungchai Park. Optimal Tire Force Allocation for Autonomous Trajectory Tracking. PhD thesis, Department of Mechanical Engineering, Standford University, USA, 2017.
[18]: Ricardo de Castro, Mara Tanelli, Rui Esteves Araújo, and Sergio Matteo Savaresi. Minimum-time path- following for highly redundant electric vehicles. IEEE Transactions on Control Systems Technology, 24(2):487–501, 2016.

Estimating the Extent of OTU in the KA Works using Indicators

3 Indicators of OTU

Wheel accelerations

$||a_1||_2 = ||a_2||_2 = ||a_3||_2 = ||a_4||_2 = ||a_{\min}||_2$

□ $\mathcal{K}_i = \frac{\sqrt{F_{xi}^2+F_{yi}^2}}{\mu_{\max}F_{zi}} = \frac{m_i}{\mu_{\max}m_i g}\sqrt{a_{xi}^2+a_{yi}^2} = \frac{1}{\mu_{\max}g}||a_i||_2$

Wheel combined slips

$||S_1||_2 = ||S_2||_2 = ||S_3||_2 = ||S_4||_2 = ||S_{\min}||_2$

□ $\mathcal{K}_i = \frac{\sqrt{F_{xi}^2+F_{yi}^2}}{\mu_{\max}F_{zi}} = \frac{k_{si}\mu_{\text{res}}(||S_i||,\chi)}{\mu_{\max}}$

Wheel longitudinal slip and slip angles

$|\kappa_1| = |\kappa_2| = |\kappa_3| = |\kappa_4| = |\kappa_{\min}|$

$|\alpha_1| = |\alpha_2| = |\alpha_3| = |\alpha_4| = |\alpha_{\min}|$

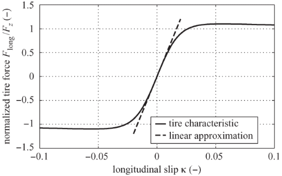


Figure: Tire characteristics, taken from [16]

■ Estimate of OTU in Ploeg et. al. [16]

□ Ploeg et. al. [16] reports:

Drive torque distribution of wheels

$\propto$

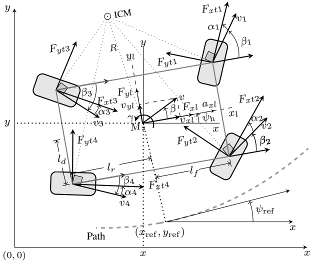
Their vertical load distribution.

$\downarrow$

$|\kappa_1| = |\kappa_2| = |\kappa_3| = |\kappa_4|$

□ All the α_i needs to generate normal tire forces towards center of rotation, they need to have same sign and $\alpha_1 \approx \alpha_2 \approx \alpha_3 \approx \alpha_4$.

■ **Extent of achievement of OTU is extremely high.**



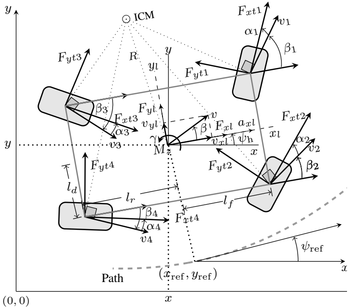
Continued...

☐ Estimate of OTU in Potluri et. al. [15]:

Simplifying assumption: $\beta = 0$

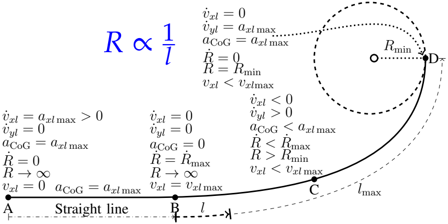
$$||a_i||_2 = \sqrt{\underbrace{k_{1i}a_{\text{CoG}}^2}_{\text{Term1}} + \underbrace{\frac{\dot{R}v_{xl}k_2}{R}\left(\frac{v_{xl}k_2}{Rk_{1i}}\dot{R} + 2\dot{v}_{xl}\right)}_{\text{Term2}}} \quad (2)$$

where, $a_{\text{CoG}} = \sqrt{\dot{v}_{xl}^2 + \frac{v_{xl}^4}{R^2}}$, $k_{1i} = \left(1 + \frac{l_d}{R}(-1)^i\right)^2 + \left(\frac{l_r}{R}\right)^2$, $k_{2i} = \left(-(-1)^i\frac{l_d}{R} - \left(\frac{l_d}{R}\right)^2 - \left(\frac{l_r}{R}\right)^2\right)$, $R_i = R\sqrt{k_{1i}}$, $v_i = v_{xl}\sqrt{k_{1i}}$, $i = 1, ..., 4$, v_i is the magnitudes of velocity of i-th wheel position, and R is the radius of ICM.



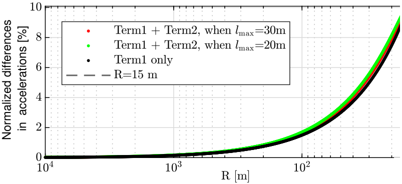
■ Extent of achievement of OTU in [15] $\geq 90\%$ for most vehicle operations

$R \propto \frac{1}{l}$



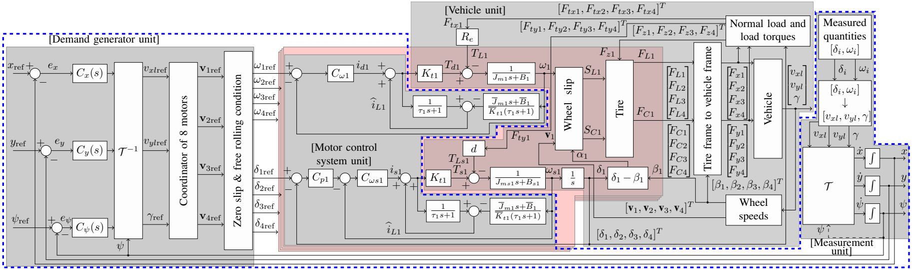
Requirement:

Motor Control System must be accurate effective.



Block Diagrammatic Examinations

■ One of the Two KA works: Potluri et al. [15]



1) Four units in gray areas, 2) Four layers in light-red shaded area, 3) Regions in blue-dotted lines.

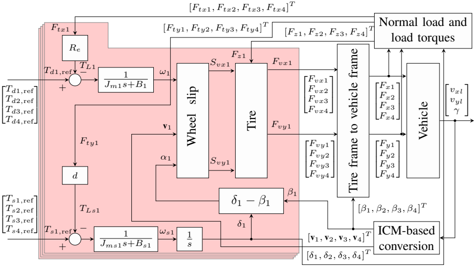


Figure: Vehicle model (Burckhardt tire model).

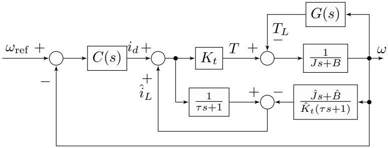
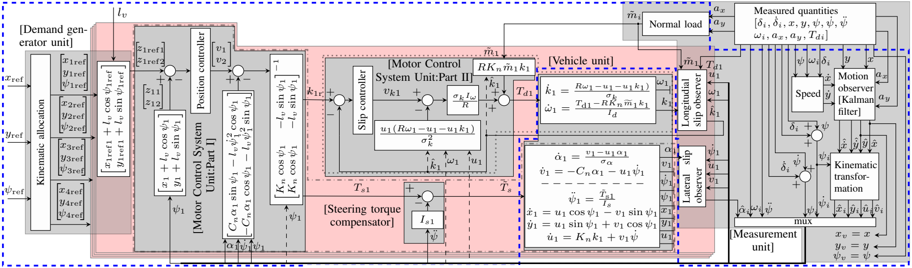


Figure: Disturbance observer-based driving motor control system.

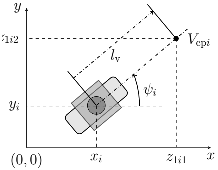
□ It uses **disturbance observer-based control** for both driving and steering motors.

Block Diagrammatic Examinations

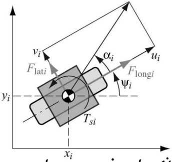
■ Remaining one KA Work: Pleog et al. [16]

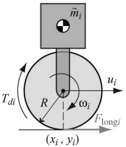


☐ **Unicycle modeling:**



$$\begin{bmatrix} z_{1i1} \\ z_{1i2} \end{bmatrix} = \begin{bmatrix} x_i + l_v \cos\psi_i \\ y_i + l_v \sin\psi_i \end{bmatrix}$$





$$\dot{x}_i = u_i \cos\psi_i - v_i \sin\psi_i \qquad \dot{y}_i = u_i \sin\psi_i + v_i \cos\psi_i$$

$$\dot{u}_i = K_n k_i + v_i \dot{\psi} \qquad \dot{v}_i = -C_n \alpha_i - u_i \dot{\psi}_i$$

$$\dot{\alpha}_i = \frac{v_i - u_i \alpha_i}{\sigma_\alpha} \qquad \dot{k}_i = \frac{R\omega_i - u_i - u_i k_i}{\sigma_k}$$

$$\ddot{\psi}_i = \frac{T_{si}}{I_s} \qquad \dot{\omega}_i = \frac{T_{di} - R K_n \tilde{m}_i k_i}{I_d}$$

☐ **9-order state:** $(x_i, u_i, y_i, v_i, \alpha_i, \psi_i, \dot{\psi}_i, k_i, \omega_i)$

☐ **Two vectors:**

1) $\mathbf{q}_i = (x_i, u_i, y_i, v_i, \alpha_i, \psi_i, \dot{\psi}_i)$

2) $\mathbf{q}_{ki} = (k_i, \omega_i)$.

☐ **Two state equations:**

$\dot{\mathbf{q}}_i = \mathbf{f}(\mathbf{q}_i) + \mathbf{G}(\mathbf{q}_i)\mathbf{u}_i$ [Part I] and

$\dot{\mathbf{q}}_{ki} = \mathbf{f}_k(\mathbf{q}_{ki}) + \mathbf{g}_k(\mathbf{q}_{ki})\mathbf{u}_{ki}$ [Part II]

Block Diagrammatic Examinations

■ Continued...

Block diagram of Part I:

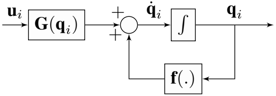


Figure A : Block diagrammatic representations of
$\dot{\mathbf{q}}_i = \mathbf{f}(\mathbf{q}_i) + \mathbf{G}(\mathbf{q}_i)\mathbf{u}_i$

Block diagram of Part II:

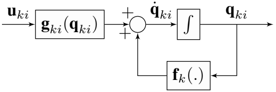


Figure B: Block diagrammatic representations of
$\dot{\mathbf{q}}_{ki} = \mathbf{f}_k(\mathbf{q}_{ki}) + \mathbf{g}_k(\mathbf{q}_{ki})\mathbf{u}_{ki}$

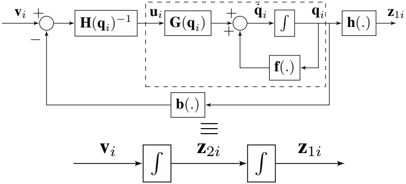


Figure C: Input-output linearization and equivalent model

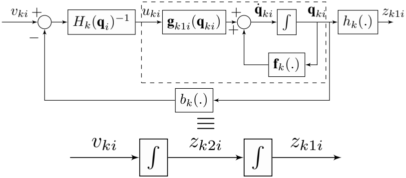


Figure D: Input-output linearization and equivalent model

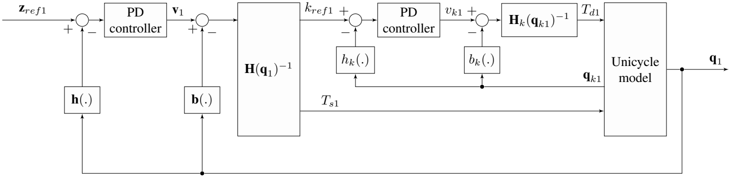
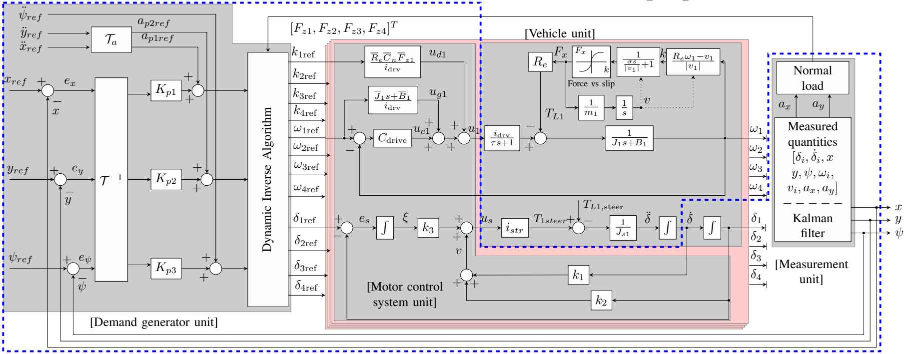


Figure E: Motor control systems for the unicycle

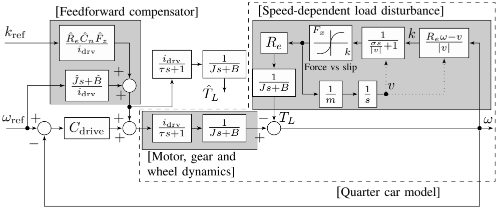
Seems to have adequate MCSs as demonstrated in [16]

Block Diagrammatic Examinations

■ One of the three FCA works: Leenen [17]



□ Driving motor control systems



□ Body acts as a speed-dependent load.

□ Wheel-speed tracking is required.

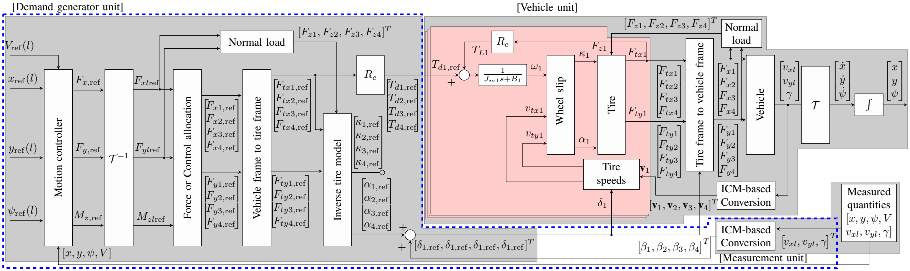
□ **Limitations in MCSs used:**

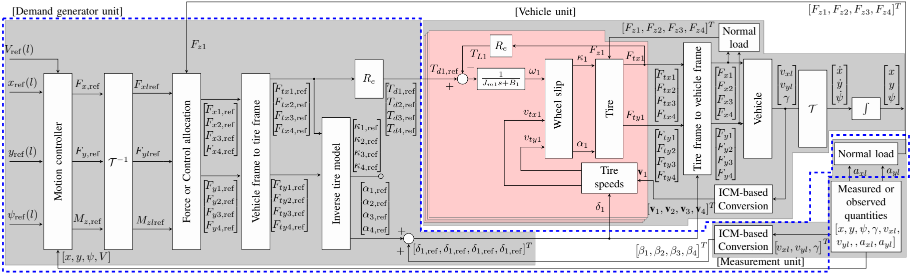
1) $\hat{T}_L$ is not real estimate of T_L

2) Transient tire dynamics is not included.

Block Diagrammatic Examinations

■ Remaining Two FCA works: Park et al. [13] and Castro et al. [18]

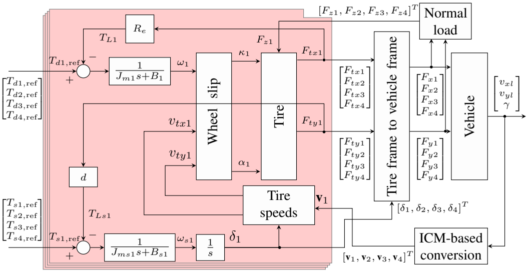




□ Motor control systems required for load-torque and steering angle tracking are not disclosed.

Block Diagrammatic Examinations

■ Block diagrammatic structure of vehicle model.

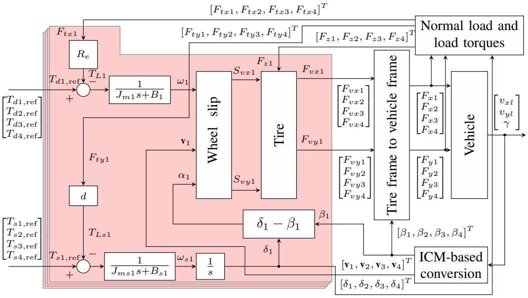


■ Vehicle model used in Park et al. [13] and Castro et al. [18].

☐ Uses Pacejka tire model.

☐ Vehicle's body acts as loads on steering and driving motors.

☐ DOBC may be used here.



■ Vehicle model used in Potluri et. al [15].

☐ Uses Burckhardt tire model.

☐ Vehicle's body acts as loads or disturbances on steering and driving motors

☐ Uses disturbance observer-based control (DOBC).

Summary of Block Diagrammatic Examinations

The 5 selected works are

Part I: Estimated the extent of achievement of OTU

KA approach

ICM allocation

Potluri et al.(IITK) [15]

Seems to have appropriate motor control systems but it requires examinations.

Position allocation

Ploeg et al. (TNO Netherlands) [16]

Have appropriate motor control systems for OTU.

FCA approach

Tire force allocation

Castro et al.(DLR) [18]

Tire force allocation

Park et al.(Stanford university) [13]

Not disclosed Motor Control Systems at slave control level

Wheel slip allocation

Leenen (Eindhoven University of Technology) [17]

Need modifications in Motor Control Systems

Suggest

Load-torque tracking

Wheel-speed tracking

NEXT: Examination of motor control systems of [15] for accuracy and disturbance rejection

Part II: Performed In-depth Block diagrammatic examinations

Motor Control Systems

Motor control systems (MCS)

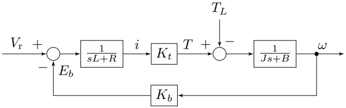
Steering MCS → does not need much attention because
- → Load torque is roughly negligible
- → Position control system for steering angle is required.
- → Mostly have high-gear ratio
- → **High-gain or PI controller can work effectively.**

Driving MCS → requires attention because
- → Load torque is speed-dependent
- → Wheel speed/torque control system is needed.
- → Mostly have low-gear ratio
- → **High-gain or PI controllers cannot be adequate for the kind of disturbances acting on the motor as discussed next.**

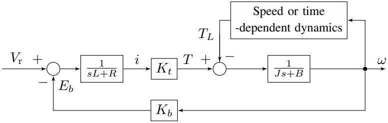
Potluri et al. [15]: Motor Model and Disturbance Model

■ Block diagram of a PM DC motor:

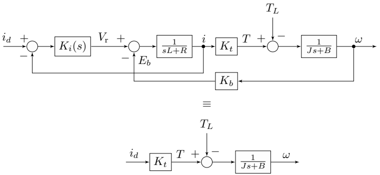
☐ Literature exclusively uses transfer function model of the motor.



☐ All the speed or time-varying dynamics appear in the feedback path from ω to T_L.

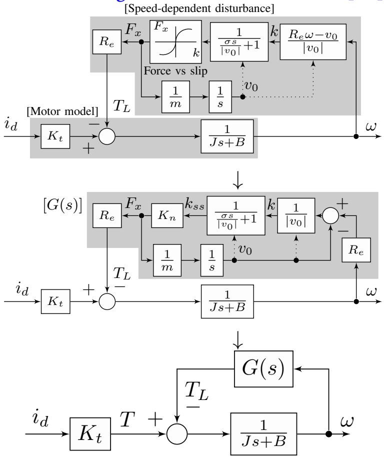


☐ When i is well-regulated, i.e. $i = i_d$



■ Speed-dependent disturbance model:

☐ Considering disturbance model from [17]:



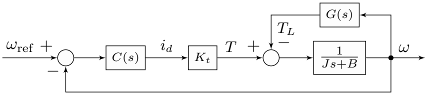
↓

$$G(s) = \frac{R_e^2 K_n s}{\sigma s^2 + |v_0| s + \frac{K_n}{m}} [19]$$

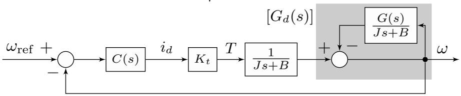
[19] R. Potluri, P. Bhole, and V. Abhishek, "Disturbance observer for multivariate speed-dependent disturbance in DC motors," Indian Control Conference (ICC), 2015.

Potluri et al. [15]: A High-Gain Control (HGC) Scheme

■ Effectiveness analysis of a high-gain control (HGC) for wheel-speed tracking



$\downarrow$



$\downarrow$

$$G_d(s) = \frac{Js+B}{Js+B+G(s)} \approx 1$$

$\downarrow$

$$\Longrightarrow \frac{(Js+B)(\sigma s^2 + |v_0|s + \frac{K_n}{m})}{(Js+B)(\sigma s^2 + |v_0|s + \frac{K_n}{m}) + R_e^2 K_n s} \approx 1$$

$\downarrow$

$$R_e^2 K_n \ll \frac{JK_n}{m} + B|v_0|$$

$\downarrow$

$$R_e^2 \ll \frac{J}{m} \Longrightarrow m \ll \frac{m_w}{2} \quad [19]$$

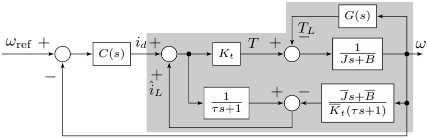
$\downarrow$

This condition cannot be satisfied, thus the HGC scheme cannot be effective.

[19] R. Potluri, P. Bhole, and V. Abhishek, “Disturbance observer for multivariate speed-dependent disturbance in DC motors,” Indian Control Conference (ICC), 2015.

Potluri et al. [12]: A Disturbance Observer-based Control (DOBC) Scheme

■ Effectiveness of a disturbance observer-based control (DOBC) scheme for wheel speed tracking at nominal values



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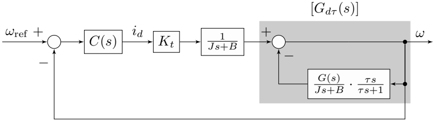


↓

$$G_{d\tau}(s) = \frac{(\sigma s^2 + |v_0|s + \frac{K_n}{m})(Js+B)(\tau s+1)}{(\sigma s^2 + |v_0|s + \frac{K_n}{m})(Js+B)(\tau s+1) + R_e^2 K_n \tau s^2} \approx 1$$

↓

$$\frac{JK_n\tau}{m} + \sigma B + (J + B\tau)|v_0| \gg R_e^2 K_n \tau$$

$$\downarrow$$

$$\implies \tau \ll \frac{\sigma B + J|v_0|}{\left(R_e^2 - \frac{J}{m}\right) K_n - B|v_0|}$$

$$\downarrow$$

$$\tau \ll \frac{\sigma B + J|v_0|}{R_e^2 K_n - B|v_0|}$$

$$\downarrow$$

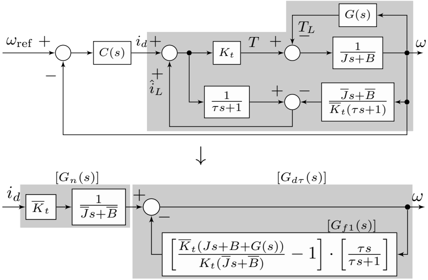
$$\tau \ll \frac{\sigma B + J|v_0|}{R_e^2 K_n} \quad [19]$$

From Table 1: $\tau \ll 2$ ms.

[19] R. Potluri, P. Bhole, and V. Abhishek, “Disturbance observer for multivariate speed-dependent disturbance in DC motors,” Indian Control Conference (ICC), 2015.

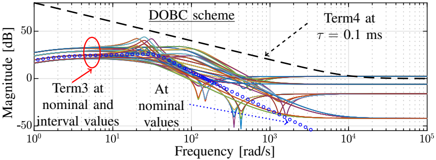
Potluri et al. [15]: A Disturbance Observer-based Control (DOBC) Scheme

■ Effectiveness of the DOBC Scheme under Parametric Uncertainty



$$\underbrace{\left|\frac{\overline{K}_t(Jj\omega + B + G(j\omega))}{K_t(\overline{J}j\omega + \overline{B})} - 1\right|}_{\text{Term3}} \ll \underbrace{\left|\frac{\tau j\omega + 1}{\tau j\omega}\right|}_{\text{Term4}}$$

Bode magnitude plots for nominal and interval values of parameters.



Above figure gives $\omega(s)/i_d(s) \approx G_n(s)$ when $G_{d\tau}(s) \approx 1$. For $G_{d\tau}(s) \approx 1$, the inequality $G_{f1}(s) \ll 1$ must hold, which gives

↓

$\tau \leq$ 0.1 ms for all interval values
$\tau \leq$ 2 ms for all interval values

Table 1: Parameter of quarter car, wheel, and IWM

Parameters	Nominal values	Interval values
K_t [N-m/A]	5.0 [15]	3.0 – 7.0
J [kg-m2]	0.36 [17]	0.25 – 0.50
B [N-m/(rad/s)]	0.57 [17]	0.30 – 0.80
K_n [N/slip]	30000	21820 – 34194 [20]
m [kg]	122 [17]	100 – 150
σ [m]	0.225	0.03 – 0.42 [17]
v_0 [m/s]	8.0	2 – 16 [15]
R_e [N-m/(rad/s)]	0.2246 [17]	0.2 – 0.25

Potluri et al. [15]: An Active Disturbance Rejection Control (ADRC) Scheme

■ Effectiveness of an ADRC Scheme under Parameter Uncertainty

$$\omega(s) = \frac{K_t}{Js+B} i_d(s) - \frac{1}{Js+B} T_L(s)$$

$$\downarrow$$

$$\dot{\omega}(t) = \frac{K_t}{J} i_d(t) - \frac{B}{J}\omega(t) - \frac{1}{J} T_L(t)$$

$$\downarrow$$

$$\dot{\omega}(t) = \underbrace{\left(-\frac{B}{J}\omega(t) - \frac{1}{J}T_L(t) - \Delta b \cdot i_d(t)\right)}_{\text{generalized disturbance f(t)}} + b_0 \cdot i_d(t) = u_0(t)$$

$$\downarrow$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}b_0\\ 0\end{bmatrix} i_d(t) + \begin{bmatrix}0\\ 1\end{bmatrix}\dot{f}(t)$$

$$[y] = [1 \quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

where $x_1(t) = \omega(t)$ and $x_2(t) = f(t)$.

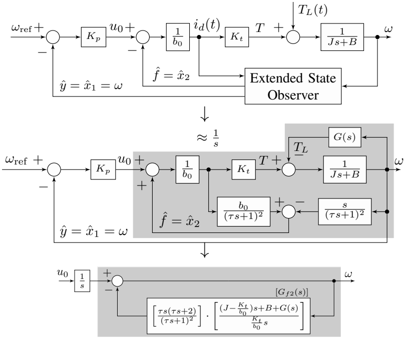
□ The equation of the Luenberger observer is:

$$\begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix} + \begin{bmatrix}b_0\\ 0\end{bmatrix} i_d(t) + \begin{bmatrix}l_1\\ l_2\end{bmatrix}(y(t) - \hat{x}_1(t))$$

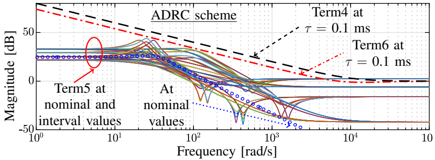
□ When placing both the poles of the observer at $s = p_o$; $p_o < 0$, $l_1 = -2p_o$ and $l_2 = p_o^2$.

□ To convert the observer in s-domain, assume $|p_0| = \frac{1}{\tau}$ in **time-constant form.**

□ $\frac{1}{K_p}$ is time constant of the closed loop system.



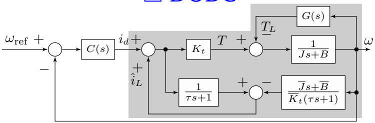
$$\underbrace{\left|\frac{(J-\frac{K_t}{b_0})j\omega + B + G(j\omega)}{\frac{K_t}{b_0}j\omega}\right|}_{\text{Term5}} \ll \underbrace{\left|\frac{(\tau j\omega+1)^2}{\tau j\omega(\tau j\omega+2)}\right|}_{\text{Term6}}$$



Potluri et al. [15]: Comparison of DOBC and ADRC Schemes

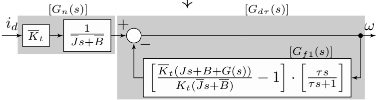
A Comparison of the DOBC and ADRC Schemes

DOBC



First order low-pass filter

↓

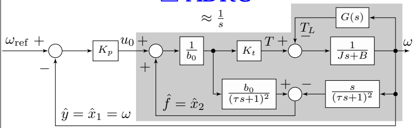


↓

$$\frac{\omega(s)}{i_d(s)} \approx G_n(s)$$

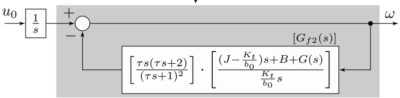
Estimate T_L as $\hat{T}_L (= \hat{i}_L \overline{K}_t)$

ADRC



Second order low-pass filter

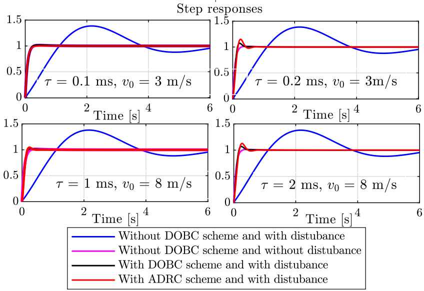
↓



↓

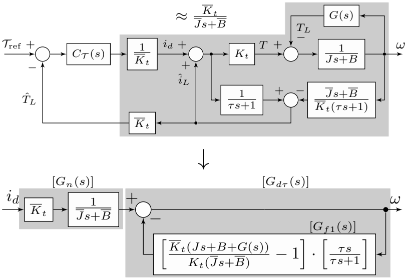
$$\frac{\omega(s)}{u_0(s)} \approx \frac{1}{s}$$

Does not estimate of T_L as $\hat{T}_L$



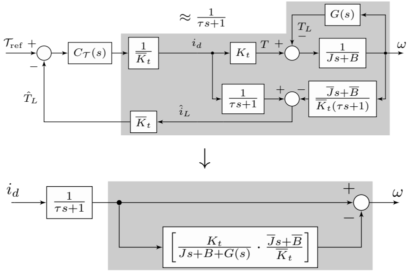
Load-torque Tracking Schemes for Park et al. [13] and Castro et al. [18]

☐ Structure 1

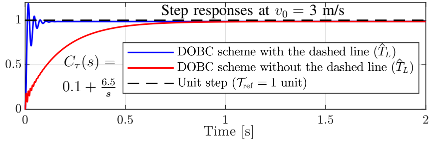


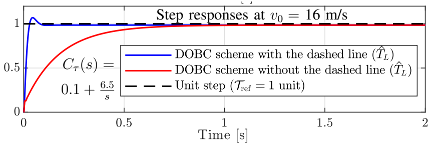
$$\underbrace{\left|\frac{\overline{K}_t(\overline{J}j\omega + B + G(j\omega))}{K_t(\overline{J}j\omega + \overline{B})} - 1\right|}_{\text{Term3}} \ll \underbrace{\left|\frac{\tau j\omega + 1}{\tau j\omega}\right|}_{\text{Term4}}$$

☐ Structure 2



$$\underbrace{\left|\frac{K_t}{Jj\omega + B + G(j\omega)}\right|}_{\text{Term7}} \ll \underbrace{\left|\frac{\overline{K}_t}{\overline{J}j\omega + \overline{B}}\right|}_{\text{Term8}}$$



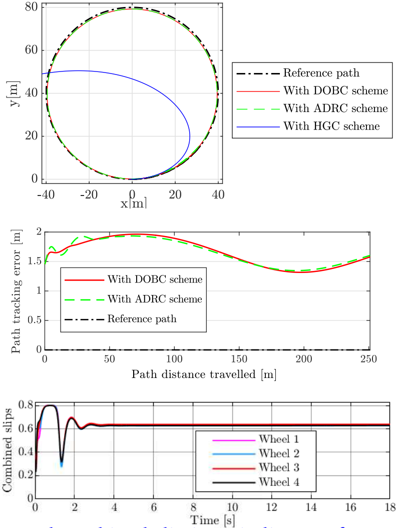


Summary and Results

■ So far:

- What is OTU?
- Importance of OTU
- Estimate the extent of OTU in the two KA works
- Block diagrammatic examination of PTC algorithms of 5 works
- Effectiveness of HGC, DOBC, and ADRC schemes for wheel speed tracking. **The last two schemes can be used in Leenen [17].**
- Effectiveness of two suggested control structures for load-torque tracking. **These structures can be used in [13] and [18], and other works too.**

■ Employing the DOBC and ADRC schemes in the PTC algorithm of [15] during circular path-tracking.



Equal combined slip is an indicator of OTU.

Key Findings and Usefulnesses of the Thesis

■ Key findings:

Optimal Tire Usage

□ Identification that most of the literature on PTC of 4WIS4WID EV aspire for OTU.

□ Importance of OTU in a PTC algorithm and its four indicators.

□ Concept of vehicle stability margin in an EV.

□ Indirect route for examining OTU in the KA works.

Block Diagrammatic Examination

□ A full PTC algorithm requires attention to both master and slave control levels.

□ The FCA works too involve kinematic calculations at the slave control levels.

□ Unlike MCS of [16], MCS of [15] is relatively simple because it treat vehicle and tire dynamics as load disturbances in natural feedback loops.

□ Practical implementation of PTC of [15] is simpler than that of the remaining four.

□ All the existing and emerging PTC algorithms likely to have a similar block diagrammatic structure with four units. However, there could be variety in each unit.

Motor Control Systems

□ A way to perform an effectiveness analysis of a DOBC scheme under parametric uncertainty during wheel-speed tracking.

□ A way to compare the DOBC scheme with an ADRC scheme in s-domain.

□ A DOBC scheme is also effective in the load-torque tracking MCSs.

Key Findings and Usefulnesses of the Thesis

Usefulnesses:

Optimal Tire Usage

- Need to be considered while developing the PTC algorithm of a 4WIS4WID EV.

Block Diagrammatic Examination

- To identify missing blocks or weaknesses in a PTC algorithm and providing appropriate solutions to them.
- To make it easy for the readers in understanding, criticizing, designing, developing, improving, and implementing a PTC algorithm.
- Block diagrams of the vehicle unit developed for Pacejka and Burckhardt tire models could be useful in developing simulation models.
- The examination suggests to use DOBC scheme while proposing MCSs.

Motor Control Systems

- The proposed DOBC and ADRC schemes demonstrates OTU in one PTC scheme.
- The proposed MCSs are applicable to these reference commands:
  - wheel-speeds and steering angles,
  - longitudinal and lateral tire forces,
  - longitudinal tire forces and steering angles,
  - driving wheel torques and steering torques,
  - driving wheel torques and steering angles

Outline

Conclusions of the Thesis

On Energy Usage

- ☐ Calculation of TTW Energy Efficiency of BEVs.
- ☐ Comparison of WTW Efficiency of IWM BEVs, SMD BEVs, ICE Vehicles

On Tire Usage

- ☐ Optimal Tire Usage
- ☐ Block Diagrammatic Examinations
- ☐ Motor Control Systems

Thank you!

Publications

On Energy Usage:

Paper 1: **Dileep Kumar**, Vasu Jain, and Ramprasad Potluri, "Energy efficiency of battery electric vehicles with in-wheel motors," SAE International Journal of Sustainable Transportation, Energy, Environment, & Policy, vol. 4, no. 13-04-01-0002, 2022.

On Tire Usage:

Paper 2: **Dileep Kumar** and Ramprasad Potluri. "Significance of Motor Control Systems for Optimal Tire Usage in 4WIS4WID Electric Vehicles". [To be submitted soon to the IEEE Transactions on Intelligent Transportation Systems for Publication.]

■ Calculation for $||a_i||_2$ at i-th wheel position

In terms of v_i, and radius of ICM of i-th wheel R_i, $||a_i||_2$ is given by:

$$||a_i||_2 = \sqrt{\dot{v}_i^2 + \left(\frac{v_i^2}{R_i}\right)} \quad (3)$$

Considering the simplifying assumption of $\beta = 0$, R_i is:

$$\begin{aligned} R_i &= \sqrt{(R + l_d(-1)^i)^2 + l_r^2} \\ &= R\sqrt{\left(1 + \frac{l_d}{R}(-1)^i\right)^2 + \left(\frac{l_r}{R}\right)^2} \\ &= R\sqrt{k_{1i}} \end{aligned} \quad (4)$$

where, $k_{1i} = \left(1 + \frac{l_d}{R}(-1)^i\right)^2 + \left(\frac{l_r}{R}\right)^2$; $i = 1 - 4$.
v_i is given by:

$$\begin{aligned} v_i &= \sqrt{\left(v_{xl} + (-1)^i \frac{v_{xl}}{R} l_d\right)^2 + \left(\frac{v_{xl}}{R} l_r\right)^2} \\ &= v_{xl}\sqrt{\left(1 + (-1)^i \frac{l_d}{R}\right)^2 + \left(\frac{l_r}{R}\right)^2} \\ &= v_{xl}\sqrt{k_{1i}} \end{aligned} \quad (5)$$

Appendix

Squaring (5) gives:

$$\begin{aligned} v_i^2 =& v_{xl}^2 k_{1i} \\ =& v_{xl}^2 \left[\left(1 + (-1)^i \frac{l_d}{R}\right)^2 + \left(\frac{l_r}{R}\right)^2 \right] \end{aligned} \tag{6}$$

Differentiating (6) with respect to time gives:

$$\begin{aligned} v_1 \dot{v}_i &= v_{xl} \dot{v}_{xl} \left[\left(1 + (-1)^i \frac{l_d}{R}\right)^2 + \left(\frac{l_r}{R}\right)^2 \right] \\ &+ \frac{v_{xl}^2}{R} \left[-(-1)^i \frac{l_d}{R} - \left(\frac{l_d}{R}\right)^2 - \left(\frac{l_r}{R}\right)^2 \right] \dot{R} \\ &= v_{xl} \dot{v}_{xl} k_{1i} + \frac{v_{xl}^2}{R} k_{2i} \dot{R} \end{aligned} \tag{7}$$

where $k_{2i} = \left[-(-1)^i \frac{l_d}{R} - \left(\frac{l_d}{R}\right)^2 - \left(\frac{l_r}{R}\right)^2 \right]$

Plugging v_i from (5), (7) gives:

$$\begin{aligned} \dot{v}_i =& \frac{v_{xl} \dot{v}_{xl} k_{1i} + \frac{v_{xl}^2}{R} k_{2i} \dot{R}}{v_{xl} \sqrt{k_{1i}}} \\ =& \frac{\dot{v}_{xl} k_{1i} + v_{xl} k_{2i} \frac{\dot{R}}{R}}{\sqrt{k_{1i}}} \end{aligned} \tag{8}$$

Squaring (8):

$$\dot{v}_i^2 = \dot{v}_{xl}^2 k_{1i} + \frac{2v_{xl}\dot{v}_{xl}k_{2i}\dot{R}}{R} + \frac{v_{xl}^2 k_{2i}^2 \dot{R}^2}{k_{1i}R^2} \tag{9}$$

Dividing (9) by (4):

$$\frac{v_i^2}{R_i} = \frac{v_{xl}^2 k_{1i}}{R\sqrt{k_{1i}}} = \frac{v_{xl}^2\sqrt{k_{1i}}}{R} \tag{10}$$

Using (9) and (10), (3) gives:

$$\begin{aligned} ||a_i||_2 &= \sqrt{\dot{v}_i^2 + \left(\frac{v_i^2}{R_i}\right)^2} \\ &= \sqrt{\dot{v}_{xl}^2 k_{1i} + \frac{2v_{xl}\dot{v}_{xl}k_{2i}\dot{R}}{R} + \frac{v_{xl}^2 k_{2i}^2\dot{R}^2}{k_{1i}} + \frac{v_{xl}^4 k_{1i}}{R^2}} \\ &= \sqrt{k_{1i}\left[\dot{v}_{xl}^2 + \frac{v_{xl}^4}{R^2}\right] + \frac{2v_{xl}\dot{v}_{xl}k_{2i}\dot{R}}{R} + \frac{v_{xl}^2 k_{2i}^2\dot{R}^2}{k_{1i}R^2}} \end{aligned} \tag{11}$$

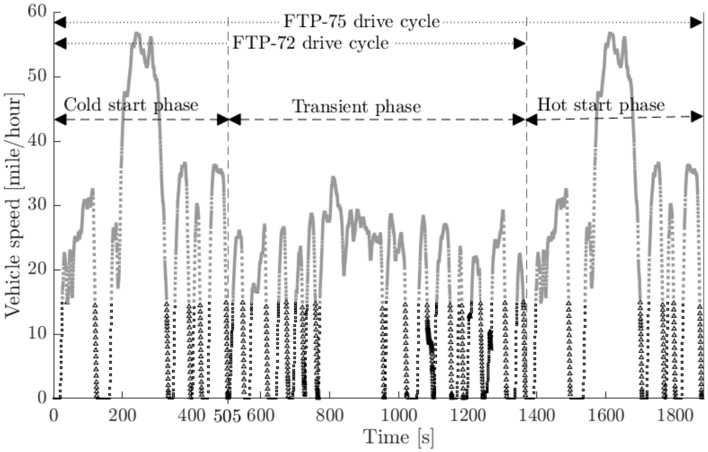
Assuming $a_{\text{CoG}} = \sqrt{\dot{v}_{xl}^2 + \frac{v_{xl}^4}{R^2}}$ as the instantaneous acceleration at the CoG of the EV, (11) gives:

$$||a_i||_2 = \sqrt{k_{1i}a_{\text{CoG}}^2 + \frac{\dot{R}v_{xl}k_{2i}}{R}\left(\frac{v_{xl}k_{2i}}{Rk_{1i}}\dot{R} + 2\dot{v}_{xl}\right)} \tag{12}$$

where, $k_{1i} = \left(1 + \frac{l_d}{R}(-1)^i\right)^2 + \left(\frac{l_r}{R}\right)^2$ and

$k_{2i} = \left(-(-1)^i\frac{l_d}{R} - \left(\frac{l_d}{R}\right)^2 - \left(\frac{l_r}{R}\right)^2\right)$.

Power Efficiency ≈ Energy Efficiency?



Stages such as motor, motor controller, and mechanical transmission do not store energy, power efficiency for these stages are defined.

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